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Quantitative Problems

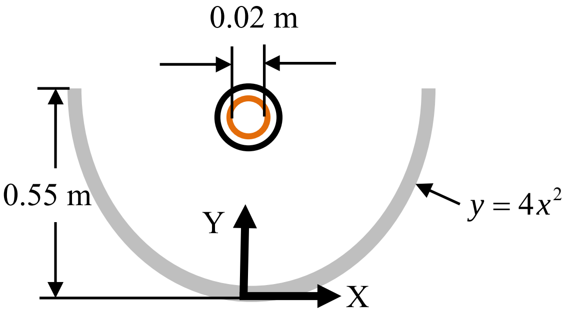
Complete Before the Midterm Exam: February 12, 2008 @ 11am

Problem 1: A Trough-Style Solar Collector

A linear, parabolic-trough-style solar-thermal energy concentrator is designed with a collector profile of the following equation:

$$y = 4x^2$$

The height (Y) of the trough is 0.55 meters, and the length (Z) of the collector is 30 meters. The concentrator surface is painted with a highly-reflective coating capable of reflecting 90% of the solar spectrum. Located at the trough's parabolic focus is the concentrator's absorber, which is a cylindrical tube (completely smooth on the inside) carrying liquid water as the working fluid. The tube's inner diameter is 2 cm, and working fluid flows through at 0.87 m/s. The absorber is painted with a black coating capable of absorbing 95% of the solar spectrum. Surrounding the tube is a glass covering, which passes 100% of the solar spectrum inward but blocks all infrared radiation outward. This glass covering is also an excellent thermal insulator, and its presence allows the absorber to be treated as adiabatic with respect to conduction and convection to the ambient environment.



A schematic representation (not to scale) showing the cross-sectional arrangement of the linear, parabolic-trough-style solar-thermal energy concentrator.

The trough collector is positioned facing east-west and its opening is rotated southward to maximize daily solar exposure. However, at the particular day and time of interest to this problem, the incident insolation arrives normal to the collector's opening at a rate of 1300 W/m^2.

1) If the working fluid inlet temperature in the concentrator's absorber tube is 300 K, what is the temperature of fluid leaving the tube 30 meters downstream?

The heated water is utilized to warm the hot end of a Stirling engine capable of operating at 40% of the Carnot efficiency. The working fluid enters a heat exchanger on the hot side of the Stirling engine at the temperature calculated in Part C.1, and it leaves the heat exchanger at 330 K. Assume the Stirling's hot end operates at the average of the heat exchanger's input and output temperatures. The cold end of the Stirling engine radiates to the ambient environment and is fixed at 305 K. The pressure drop across the heat exchanger is 60,000 Pa.

2) If the pump used to move working fluid through the system is 75% efficient, calculate the total pumping power required to move the water through the concentrator's absorber and the heat exchanger.

3) What is the total work output of the combined solar-thermal energy concentrator and Stirling engine taking into account the parasitic impact of the pump?

4) Given that conventional photovoltaic cells can achieve solar photon to electron conversion efficiencies better than 10%, would it be more efficient to replace the concentrator's thermal absorber and Stirling engine with a linear array of PV panels situated at the focus of the parabolic concentrator?

Problem 2: Cooling Tower Operation

The function of a wet cooling tower is to remove the entropy from hot water via direct contact with the air. One tower configuration of interest is a rectangular channel with width W= 10 cm that is long enough that W << L. Liquid water flows vertically downward by gravity as a thin film over each of the channel's internal surfaces. Air flows vertically upward in the counter-flow direction. Due to evaporation, some liquid water is lost into the air during the cooling process.

The following parameters may be important: the air flow Reynolds number, Re = 50,000; the temperatures at the inlet (station 1) are $T_{air1} = 30$ °C and $T_{water1} = 50$ °C; and the latent heat of evaporation for water, $h_{fg} = 2.54 \times 10^6$ J/kg.

Properties of air that may be important include: c_p= 1005 J/kg-K; $\rho = 1.165$ kg/m^3; $\nu = 15.66 \times 10^{-6}$ m^2/s; Pr = 0.69; Schmidt number (for water vapor diffusion in air), Sc = 0.6; and the relative humidity of the inlet air is 20%.

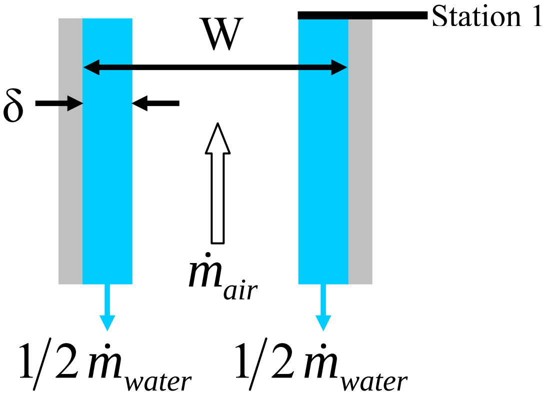
Assume the following: 1) the air flow is fully developed; 2) for the water layer $\delta/W << 1$; 3) the water flow is laminar; and 4) inertial forces are negligible. When calculating interface shear and heat and mass transfer coefficients, assume that the water velocity is negligible compared to the air velocity.

1) Find an expression for the velocity distribution in the water layer.

2) From the expression in (1), find the local liquid water mass flow rate (per unit channel depth) as a function of the (local) layer thickness, δ, air flow Re-number, channel width (W), and pertinent properties of water and air.

3) Estimate the heat and mass transfer coefficients at the liquid/air interface.

4) Explain qualitatively in a sentence or two whether the water in this system can be cooled below the air inlet temperature (30 °C).



A schematic representation (not to scale) of water and air flows within the rectangular cooling tower.

Problem 3: Solar Powered Air Conditioner

A building is designed with a solar-thermal air conditioning system that lifts 300 kW while maintaining the space at 300 K. Solar insolation is 1000 w/m^2, and solar-thermal energy is collected on the roof. For the time period of interest, the effective hot reservoir temperature is a constant 400 K. Heat is rejected to a deep-soil thermal sink that maintains a constant effective temperature of 310 K.

1) Develop the most efficient thermodynamic schematic design for the air conditioning system described above. You may assume that any cyclic device used in your design is reversible.

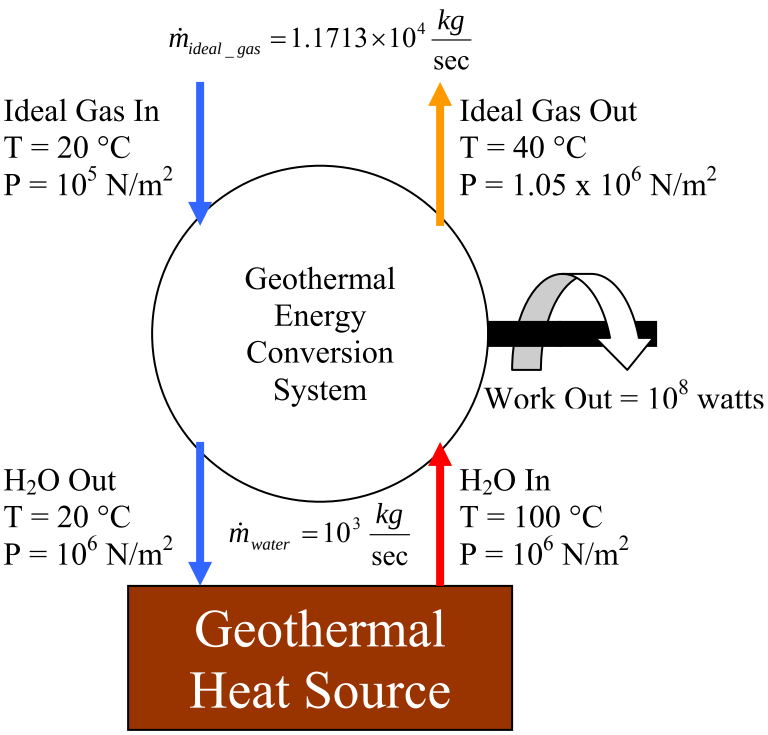
2) Estimate the minimum solar collection area required to make the system described above operate during the period of interest.

Problem 4: Geothermal Energy Conversion System

You are an engineer retained as a consultant for a venture capital firm to evaluate the potential of energy technology companies. The firm is approached by a start-up company that asks for funding to support a new geothermal energy conversion system that they have patented. The system performs in steady state according to the schematic to the right. How do you advise the venture capital firm to proceed and why?

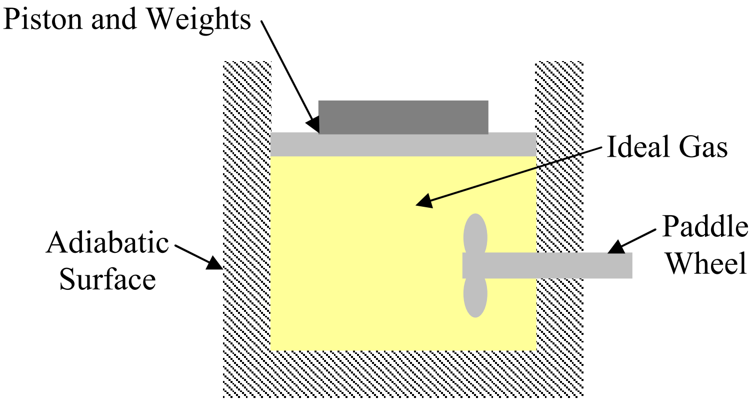
The following ideal gas property values may be useful: R = 287 J/Kg-K; c_v = 716 K/kg-K .

The following liquid water properties may be useful: c = 4187 J/kg-K; ρ = 1000 kg/m^3 .



A schematic representation of the proposed geothermal energy conversion system under scrutiny.

Problem 5: Basic Energy and Entropy Calculations via an Unconventional Calorimeter



A schematic representation of the calorimeter with integrated piston-weight boundary.

Ten grams of air are confined in a piston-cylinder apparatus whose outside surfaces are adiabatic. Inserted in the gas through the cylinder wall is a paddle wheel, which is attached to a shaft. The shaft and the paddle wheel are fabricated from a material whose thermal conductivity is zero. In the initial equilibrium state of this system, the air, the piston cylinder apparatus, and the paddle wheel are in a state of thermal and mechanical equilibrium at a pressure of $P = 10^5$ N/m^2 and a temperature of 300 K. The shaft is set rotating for a brief period of time and then stopped. After the system has reached a new equilibrium state, it is found that the piston is displaced upward a distance such that the volume has increased by 10 percent.

The air can be modeled as an ideal gas with R = 287 J/kg-K and c_v = 716 J/kg-K, and the piston cylinder apparatus can be modeled as a pure thermal system with $C = 10^3$ J/kg-K.

1) What is the temperature and volume of the air in the new equilibrium state?
2) Calculate the work transfer interaction between the air and the paddle wheel.
3) Calculate the heat transfer experienced by the air.
4) Calculate the changes in entropy for the air and this piston-cylinder apparatus.
5) What is the entropy transfer from the air and for the piston-cylinder apparatus?
6) Is the work transfer process between the paddle wheel and the air reversible? Why?
7) Is the heat transfer process between the air and the piston-cylinder apparatus reversible? Why?

Problem 6: Energy Storage using Compressed Air

In industrial facilities, compressed air is sometimes referred to as "the third" utility after lighting and HVAC. In these facilities, compressed air energy storage systems have been utilized for energy shaving during peak periods to reduce electricity costs. Compressed air is generated and then stored in a high-pressure reservoir during off-peak periods. When a peak energy demand period hits, the stored high-pressure air is then bled through a turbine to generate supplemental electricity.

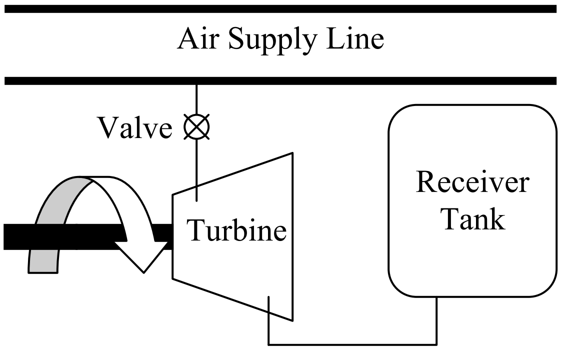
One such system is shown schematically below. During off-peak periods, the air supply line can supply any mass of air at a constant pressure of $P_{supply} = 10^6$ N/m^2 and a temperature of 300 K. The supply line is connected to a reversible, adiabatic turbine of negligible internal volume that discharges into a receiver having a volume, $V_r = 10^3$ m^3.

In the off-peak electricity demand period, the internal volume of the turbine and receiver are evacuated. The valve separating the air supply line from the turbine is opened and air flows from the supply line through the turbine into the receiver. When the pressure of the air in the receiver reaches the pressure of the supply line, the valve is closed to store the air for later energy extraction.

1) What is the mass of air, m_{air}, in the receiver when the valve is closed?
2) What is the temperature of the air in the receiver when the valve is closed?
3) How much shaft work does the turbine produce?

In a peak electricity demand period, the air supply line is shut down and evacuated. The valve separating the air supply line from the turbine is opened and air flows from the receiver through the turbine and into the supply line. You may assume that at the beginning of this process, the air in the receiver tank is at the same temperature as calculated in Part (2). You may also assume that the turbine operates reversibly and adiabatically at full efficiency regardless of which direction air traverses through it.

4) How much shaft work does the turbine produce (in other words, how much energy was stored)?
5) What is the minimum amount of work that had to be done by the building air compressor system in an off-peak demand period to store the energy calculated in Part (4)?



Schematic of compressed air energy storage scheme system for peak shaving.

Problem 7: Wax-based Phase Change Thermal Energy Storage System

A solar energy thermal storage system uses the phase change of wax as a heat storage medium. During the day, the wax is melted using solar heat; during the evening, the molten max is used to heat water, re-solidifying the wax the process. The resulting warm water is distributed through a building to heat it. However, for this problem, we focus on the process by which the wax heats the water.

The wax is encapsulated in a number of elastic spherical bladders, each 2.6 cm in diameter. The wall thickness of the bladders in 3mm with a thermal conductivity, $k_{bladder}$ = 0.3 W/m-K. For this problem, ignore the volume change that accompanies melting. Properties of the solid wax are given in the following table

Properties of Wax

Melting Point	Latent Heat of Fusion	Thermal Conductivity	Specific Heat Capacity	Density	Thermal Diffusivity
[°C]	[kJ/kg]	[W/m-K]	[J/kg-K]	[kg/m^3]	[m^2/s]
37	247	0.23	2068	783	9.23X10^{-8}

You may assume that liquid wax properties are similar to the solid properties, but that the liquid was has a low viscosity.

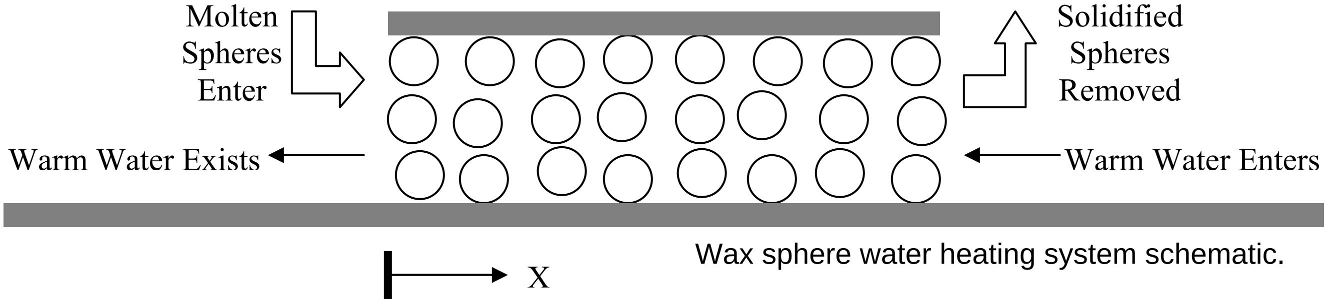
When heating in the building is desired, the spheres are drawn through a stream of water, moving against the current. The spheres are held in bundles by a net and the bundles are passed through the water channel by a mechanism. Details of the netting and the mechanism are unimportant. Effectively, the channel is filled with spheres leaving a void fraction of 50%, and the water flows through the spaces between the spheres. The channel has a square cross section, 8 cm on a side. The length of the channel is not yet specified.

Water enters the channel with a bulk temperature of 20 °C and exists with a bulk temperature of 35 °C. The mass flow rate of water is m = 0.0500 kg/s. The wax spheres enter at a temperature of 45 °C and the wax inside circulates freely (i.e., the wax is uniform in temperature inside each bladder). The effective mass flow rate of the wax is 0.0115 kg/s. The mean velocity of the wax relative to the spheres within the sphere filled channel is 0.020 m/s.

1) Estimate the heat transfer coefficient between the spheres and the water.

2) Find the temperature of the water at the point where the wax begins to freeze and find the distance from the inlet at which this occurs.

3. Estimate the additional distance along the channel that is required to fully solidify the wax. For simplicity, you may take the water temperature to be constant at 27 °C at the point where the wax has solidified.



Wax sphere water heating system schematic.

Problem 8: Simple Model for Solar Thermoelectric Generator

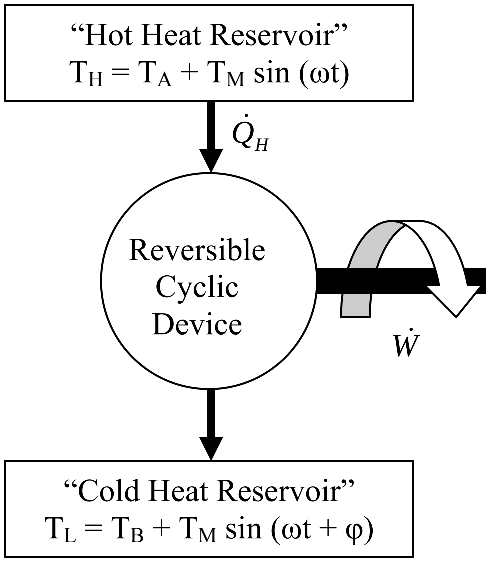
One solar-thermal energy collection technique proposed by undergraduate energy researchers at UNT is to heat the warm end of a thermoelectric generator using sunlight concentrated through a Fresnel lens while the other side of the generator is cooled via contact with a high-thermal-conductivity metallic rod embedded in the soil. The power output of this device is tied to the location of the sun in the sky as well as the temperature of the soil; both can be roughly approximated by sine functions with ranges from 0 to 2π radians. Given the thermal mass of the soil, its temperature lags behind the insolation arising from the sun by a factor of φ (where the values of φ range between 0 and 2π radians).

Approximate the thermoelectric generator as a reversible cyclic device operating between two thermal systems that can be modeled as "heat reservoirs," one at high temperature, T_H, and the second at low temperature, T_L. However, unlike true temperature reservoirs the temperatures of these "heat reservoirs" are not constant with time. Their temperatures are given by the following expressions:

$$T_H = T_A + T_M \sin \omega t \qquad T_L = T_B + T_M \sin(\omega t + \phi)$$

where ω is some constant associated with the speed of the sun across the sky. Regardless of the particular value of T_H at any instant of time, the rate of heat transfer between the high-temperature "heat reservoir" and the cyclic device is a constant Q_H. The first and second laws of thermodynamics must be satisfied at every instant of time. Since the thermoelectric generator is very small with almost no thermal mass, there is no possibility of phasing energy and entropy transport or storage within the reversible cyclic device.

1) Derive an expression for the power produced by the cyclic device in terms of the parameters used to specify the situation.
2. What is the average value of the power produced by the cyclic device?
3. What value of φ will produce the greatest peak power?
4. What value of φ will produce the greatest average power?
5. Is this model a realistic representation for the energy generation apparatus described? Why or why not?



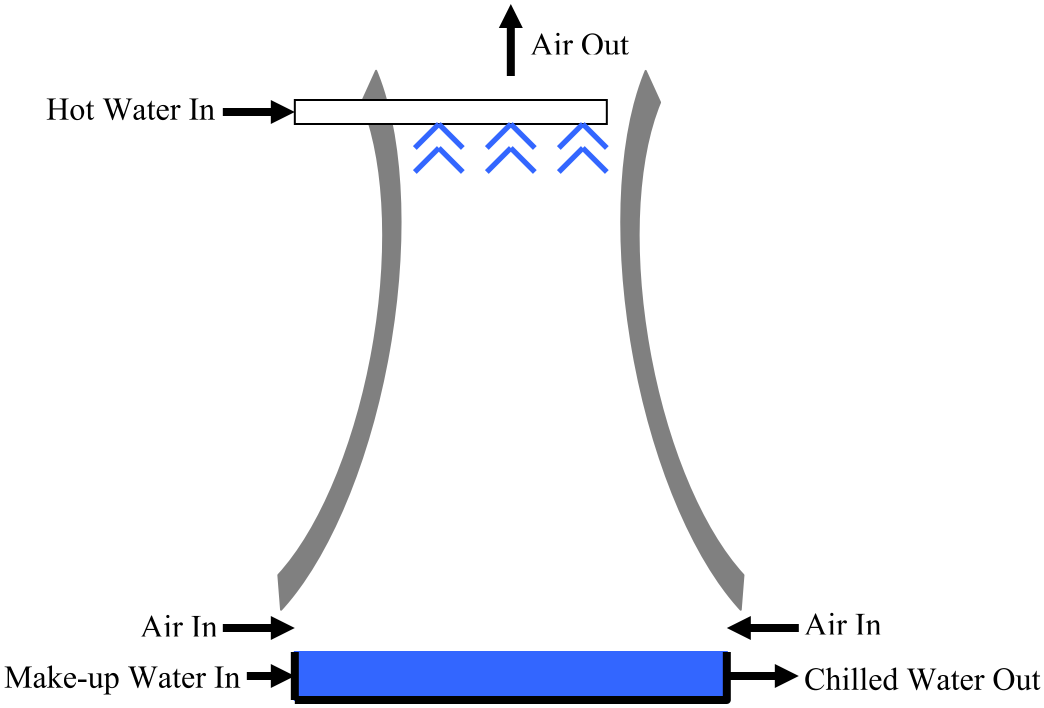
Simple representation of reversible heat engine operating between two sinusoidal, phased "heat reservoirs".

Problem 9: Energy, Entropy, and Mass Flows in a Cooling Tower

To satisfy the second law of thermodynamics, power plants must reject entropy to the environment. One common technique is to use the atmosphere as a "heat reservoir" to reject entropy through a cooling tower. In the cooling tower, hot liquid water from the power plant turbine's outlet is brought into direct contact with atmospheric air. This unsaturated air contains only a small amount of water vapor, and it is capable of absorbing more water. Some of the hot water evaporates as the air becomes saturated with water vapor. Energy is required to convert liquid water into vapor, and this energy is drawn from the remaining liquid. Hence, the temperature (and entropy content) of the remaining water in the cooling tower is reduced. The cooled liquid water is re-circulated into the power plant to begin the process again. Some water is lost into the air during evaporation, and this liquid must be replenished in the form of "Make-up Water" injected at the bottom of the cooling tower.

As shown below, hot liquid water from the power plant turbine enters near the top of a cooling tower at a temperature of $T_{water,in} = 40$ °C at a steady rate of 2×10^5 kg/hr. Liquid water leaves from the bottom of the tower at $T_{water,out} = 22$ °C. Atmospheric air enters the cooling tower from the bottom at a temperature of $T_{air,in} = 20$ °C and a relative humidity of 50 percent. Saturated air ($\Phi = 100\%$) leaves the top fof the cooling tower at a temperature of 35 °C. In your calculations, you may assume that the changes in kinetic energy and potential energy are negligible.

1) Calculate the required mass flow rate of atmospheric air to meet the inlet / outlet conditions.
2) Calculate the mass of make-up water that must be added to the cooling water stream.
3) Calculate the entropy change of the cooling water as it passes through the cooling tower.
4) If the energy conversion efficiency of the cycle that the produced the entropy in Part (3) is 38.5 percent, calculate the power produced by the power plant.



Cooling tower schematic demonstrating flows of air and water.